GCE AS/A level

0975/01

MATHEMATICS C3
Pure Mathematics

A.M. FRIDAY, 1 June 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

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1. *(a)* Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_0^1 e^{x^2}\,dx.$$

Show your working and give your answer correct to four decimal places. [4]

(b) **Use your answer to part *(a)*** to deduce an approximate value for the integral

$$\int_0^1 e^{x^2+3}\,dx.$$ [2]

2. *(a)* Show, by counter-example, that the statement

'If $\cos\theta = \cos\phi$ then $\sin\theta = \sin\phi$'

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$13\tan^2\theta = 5\sec^2\theta + 6\tan\theta.$$ [6]

3. *(a)* The curve C is defined by

$$x^3 - 4x^2y = 2y^3 - 3x - 2.$$

Find the value of $\frac{dy}{dx}$ at the point (3, 1). [4]

(b) Given that

$$x = \sin at,\ y = \cos at,$$

where a is a constant, find and simplify

(i) an expression for $\frac{dy}{dx}$ in terms of a and t,

(ii) an expression for $\frac{d^2y}{dx^2}$ in terms of a and t. [7]

4. Show that the equation

$$\cos x - 5x + 2 = 0$$

has a root α between 0 and $\frac{\pi}{4}$.

The recurrence relation

$$x_{n+1} = \frac{1}{5}(2 + \cos x_n)$$

with $x_0 = 0{\cdot}6$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4. Write down the value of x_4 correct to five decimal places and prove that this is the value of α correct to five decimal places. [7]

5. Differentiate each of the following with respect to x.

(a) $\ln(7 + 2x - 3x^2)$ *(b)* $e^{\tan x}$ *(c)* $5x^2 \sin^{-1} x$ [2], [2], [3]

6. Find

(a) (i) $\int 3e^{2-\frac{x}{4}}\,dx$ (ii) $\int \frac{9}{(2x-3)^6}\,dx$ (iii) $\int \frac{7}{3x+1}\,dx$ [6]

(b) Given that $0 < a < \frac{\pi}{2}$ and that

$$\int_0^a \sin 2x\,dx = \frac{1}{4},$$

find the value of the constant a. [5]

7. Solve the following.

(a) $4|x-3| + 2 = 8 - 5|x-3|$ [3]

(b) $|5x-2| \leqslant 3$ [3]

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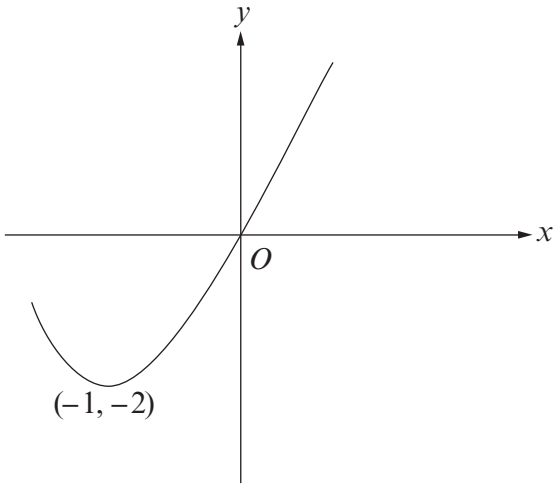
8. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the origin and has a minimum point at $(-1, -2)$.



Sketch the graph of $y = -4f(2x)$, indicating the coordinates of the stationary point. [3]

9. The function f has domain $(-\infty, 0)$ and is defined by

$$f(x) = \frac{x^2 + 3}{x^2 + 5}.$$

(a) (i) Show that $f'(x)$ is always negative.

(ii) Write down the range of f. [6]

(b) (i) Find an expression for $f^{-1}(x)$.

(ii) Write down the range and domain of f^{-1}. [5]

10. The function g has domain $(-\infty, \infty)$ and is defined by

$$g(x) = \sqrt{3x^2 + 7}.$$

Solve the equation

$$gg(x) = 8.$$ [5]